

# LOOP ALGORITHMS FOR DUCTILITY ANALYSIS OF COLUMN REINFORCED STEEL WITH YIELD STRENGTH ABOVE 500 MPA

<sup>1</sup>AHMAD FAZA AZMI

<sup>1</sup>East Java Regional Residential Infrastructure Center, Directorate General Of Copyright Works,

Minister for Public Works and Human Settlements

e-mail: [faza.azmi@pu.go.id](mailto:faza.azmi@pu.go.id)

## ABSTRACT

*There are many benefits to the use of high-strength reinforcement (above 500 MPa) in reinforced concrete buildings. The advantages of using high-strength reinforcement are reduction steel volume and dimension, reduced construction time, reduction in reinforcement congestion, as well as savings in materials and worker cost. Meanwhile, the investigation of ductility of reinforced concrete element with high-strength reinforcement to resist earthquake effects under current design procedure is needed. In the current standard, ACI 318-71, The maximum specified yield strength was restricted to 60 Ksi (413 MPa) for reinforcement in special seismic system. There were also no ASTM standard specifications for reinforcement with yield strength above 500 MPa. In the design of seismic resisting structures, the analysis of curvature ductility and flexural overstrength factor is of important consideration in order to avoid brittle failure. This paper attempts to analyze the ductility and re-evaluate the flexural overstrength factor of reinforced concrete column. The tensile tests of steel reinforcement with yield strength above 500 MPa generates stress-strain curve. An idealisations for the monotonic stress-strain curve proposed by Mander was adopted in this study. Whereas in this numerical study of confined concrete columns, the behavior of concrete core is modeled by the stress-strain relationship of confined concrete proposed by Kappos-Konstantinidis. This stress strain model was used for the moment, curvature, ductility, and flexural overstrength factor analysis.*

**Keywords:** Column, Steel, Ductility, Flexural, Overstrength factor

## 1. INTRODUCTION

There are many potential advantages to the use of high-strength reinforcement (above 500 MPa) in reinforced concrete structures. There are reduction steel volume, reduced construction time, reduction in reinforcement congestion, as well as savings in material and worker cost. Meanwhile, the investigation of ductility of reinforced concrete element with high-strength reinforcement to resist earthquake effects under current design procedure is needed. In the current standard, ACI 318-71, The maximum specified yield strength was restricted to 60 Ksi (413 MPa) for reinforcement in special seismic system. There were also no ASTM standard specifications for reinforcement with yield strength above 500 MPa.

This paper attempts to analyze the ductility and re-evaluate the flexural overstrength factor of reinforced concrete column. The stress-strain model proposed by Mander has six key parameters. The six key parameters, which are used to form the stress-strain curve, are obtained from the tensile tests, there are  $f_y, \epsilon_y, E_s, f_{sh}, \epsilon_{sh}, E_{sh}$ . The number of the samples tested is 78 specimen and the mean value of the six key parameters are listed in Table 2.

## 2. STEEL REINFORCEMENT MODELS

Previous investigations have shown that the plastic hinge behavior of reinforced concrete members is determined by the stress strain curve of the reinforcing steel (Park, 1977). The tensile tests are necessary to determine the stress strain characteristic of the reinforcing steel. The tensile tests generate stress-strain curve of the steel bars with yield strength above 500 MPa. The stress-strain curve of the steel bars, obtained from tensile tests, approach the stress-strain curve proposed by Mander.

Based on the stress-strain properties of reinforcing steel, theoretical curvature ductility, and overstrength factor analyses are carried out for reinforced concrete column.

Curvature ductility and flexural overstrength factor analysis was calculated using numerical analysis by using loop algorithms.

Numerical analysis can be used to simulate the behavior of reinforced concrete columns by entering the strain values into the given formula (function) so as to produce stress and internal forces values in the reinforced concrete column.

For this numerical analysis to be performed, a function (formula) which represents the relationship between stress and strain (steel and concrete) in a reinforced concrete element is required. While on the steel tensile test, it only produces stress-strain curve of steel without the function (formula) that forms the stress-strain curve of the steel bars.

Due to the above problems, this study adopts the stress-strain curve of the steel bars proposed by Mander so that the strain value can be entered into numerical analysis by computer program (VBA Macro Excel) so that the value of stress and internal forces in the reinforced concrete column can be obtained. Furthermore, the stress-strain curve of the steel bars proposed by Mander is considered to represent the stress-strain curve of tensile test results.

The stress-strain curve of the steel bars, proposed by Mander, is calculated using the following equation.

a. Linear Elastic ( $0 \leq \varepsilon_s \leq \varepsilon_y$ )

$$f_s = E_t \varepsilon_s \quad (1)$$

$$E_t = E_s \quad (2)$$

$$\varepsilon_y = f_s / E_s \quad (3)$$

$E_t$  = tangen modulus

$E_s$  = modulus of elasticity of the steel (Young's modulus)

b. Yield Plateau ( $\varepsilon_y < \varepsilon_s \leq \varepsilon_{sh}$ )

$$f_s = f_y, E_t = 0 \quad (4)$$

c. Strain Hardening ( $\varepsilon_{sh} < \varepsilon_s \leq \varepsilon_{su}$ )

Strain that occurs is followed by the increased value of  $f_s$  exceed  $f_y$  and continue until the ultimate strain ( $\varepsilon_{su}$ ) is reached. At point D maximum stress is reached. The expression for the strain hardening area is in the form of a power curve, with the ultimate stress-strain coordinate as origin, as follows :

$$\left[ \frac{f_{su} - f_s}{f_{su} - f_y} \right] = \left[ \frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}} \right]^P \quad (5)$$

$$f_s = f_{su} + (f_y - f_{su}) \left| \frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}} \right|^P \quad (6)$$

Where P is the strain hardening power and can be determined by differentiating Equation 6 to give the tangent modulus :

$$E_t = \frac{df_s}{d\varepsilon_s} = P \left[ \frac{f_{su} - f_y}{\varepsilon_{su} - \varepsilon_{sh}} \right] \left| \frac{\varepsilon_{su} - \varepsilon_s}{\varepsilon_{su} - \varepsilon_{sh}} \right|^{P-1} \quad (7)$$

Since the strain hardening modulus ( $E_{sh}$ ) occurs when  $\varepsilon_s = \varepsilon_{sh}$  ,therefore :

$$E_t = E_{sh} = P \left[ \frac{f_{su} - f_y}{\varepsilon_{su} - \varepsilon_{sh}} \right] \text{ atau} \quad (8)$$

$$P = E_{sh} \left[ \frac{\varepsilon_{su} - \varepsilon_{sh}}{f_{su} - f_y} \right] \quad (9)$$

Stress at yield point (point B in figure 1) is considered as yield strength, and used as parameter in elastic design of steel reinforcement. The modulus of elasticity average values (Es) is determined by the slope of the linear static. Which is generally determined as 200 GPa, however, from the tensile tests, the modulus of elasticity average values is 212288 MPa.

The comparison stress-strain value that is obtained from tensile tests and The stress-strain value that is obtained from mander formula is shown in Table 1.

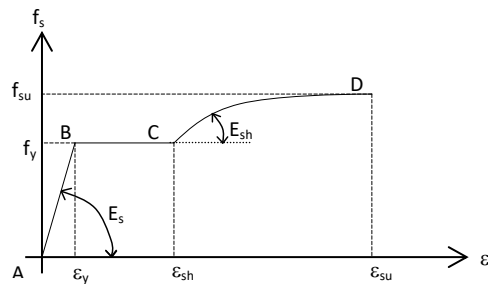


Figure 1. Stress-strain curve of steel (Mander et al, 1984)

The stress-strain value of the reinforcing steel proposed by Mander is shown in Table 1. It can be obtained from the mean values of stress and strain of reinforcing steel at yield, initial strain hardening, and ultimate strain hardening which are shown in Table 2 (Result and Discussion section), by inputting the value of strain into equation (1) to equation (9). Then the strain value is increased by certain increment.

The stress-strain value of the reinforcing steel obtained from tensile test results also shown in Table 1. It is obtained from the mean values of stress-strain test specimen.

In Figure 2, the stress-strain curve shows an explicitly upper yield strength point. The upper yield strength value, from tensile tests, as shown in Table 1, is 526.87 MPa. The relative magnitude of the upper yield point depends on the speed of testing, the shape of the section and the form of the specimen (Park and Paulay, 1975).

Table 1. Stress strain of steel from the tensile test (MPa)

Condition	Mander		Tensile Test	
	Strain	Stress	Strain	Stress
Yield	0.002441	518.1224	0.002441	518.1224
Yield Plateau	0.005383	518.1224	0.005383	526.8793
	0.008325	518.1224	0.008325	528.1957
	0.011267	518.1224	0.011267	528.6989
	0.014209	518.1224	0.014209	529.3922
Strain hardening	0.017151	518.1224	0.017151	530.5282
	0.029513	549.6992	0.029513	570.127
	0.041874	577.5556	0.041874	598.9098
	0.054235	601.7469	0.054235	621.9042
	0.066597	622.3352	0.066597	640.6962
	0.078958	639.3906	0.078958	653.4712
	0.09132	652.9946	0.09132	662.4942
	0.103681	663.2444	0.103681	668.877
	0.116042	670.2621	0.116042	672.487
	0.128404	674.2146	0.128404	674.384
Ultimate	0.140765	675.3878	0.140765	675.3878

The yield plateau length (B-C in Figure 1) is generally function of the strength of the steel. From monotonic tension tests, the stress value at yield plateau region is between 526,87 to 530,52 MPa whereas the stress value obtained from mander formula classically treated as flat and with zero tangent modulus as shown in Figure 2, the stress obtained from mander formula remains constant while the strain continues to increase. It caused the difference value of stress between stress-strain curve proposed by mander with stress-strain curve obtained from monotonic tension test although not significant. The ultimate stress occurs at Point D in Figure 1. This point is assumed as the ultimate strain rather than the fracture strain which occurs at a lower stress and higher strain. The comparison between stress- strain curve of reinforcing steel obtained from monotonic tension tests and mander formula is shown Figure 2.

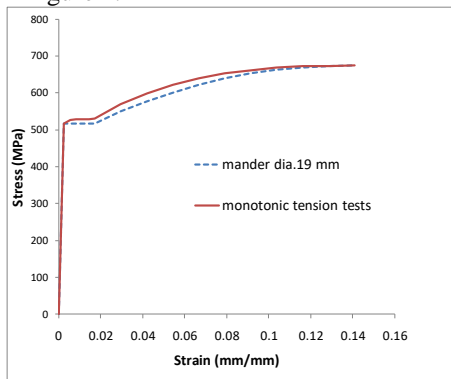


Figure 2. Comparisons Stress-Strain Curve of Reinforcing Steel between monotonic tension tests and mander

### 3. CONFINED CONCRETE MODELS

The stress-strain model proposed by Kappos-Konstantinidis for confined concrete under monotonic compressive loading was adopted. The comparison of stress-strain model between confined concrete (Kappos-Konstantinidis) and unconfined concrete (Kent-Park) shown in Figure 3. The definition of ultimate strain assumed at which ultimate stress occurs, rather than at fracture point which occurs at a lower stress. Confinement in addition to increasing stress and strain of concrete, also to avoid over-reinforced condition on reinforced concrete columns. It is necessary for the steel to be able to undergo large plastic strains before the concrete reaches the ultimate strain.

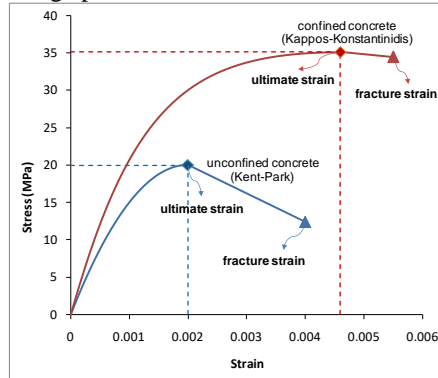


Figure 3. Confined and Unconfined Stress-Strain Curve of Concrete

### 4. MOMENT, CURVATURE, DUCTILITY AND OVERSTRENGTH FACTOR ANALYSES

The curvature of a member is defined as the rotation per unit length. The moment-curvature curve for a reinforced concrete section can be traced theoretically using the requirements of strain compatibility and equilibrium of internal forces (Park and Paulay, 1975).

The analysis start from  $\epsilon_{cm} = 0.000005$ . and then loop algorithms gradually increasing the  $\epsilon_{cm}$  value by increments of 0.000005. For each value of  $\epsilon_{cm}$  the neutral axis depth (kd) is adjusted and the internal forces in the concrete and the steel is found. When the internal forces is found, the moment M and curvature is found.

### 5. RESULT AND DISCUSSION

When The stress-strain properties of reinforcing steel obtained from a monotonic tension test as used for longitudinal reinforcement shown in Table 2. There are 30 models with various reinforced concrete column properties which are used as models in this investigation. The data value for some models of the specimen to be analyzed, can be seen in Table 3.

The moment-curvature relationship is shown in Figure 4. Figure 4 exhibit a discontinuity at first yield of the tension steel and have been terminated when the steel strain reaches strain hardening ultimate ( $\epsilon_{sh}$  is assumed as  $\epsilon_{su}$ ). Figure 4 indicate the ductility of the section is significantly reduced by the presence of axial load.

Table 2. Steel properties from the tensile test (MPa)

Diameter of Bars 19 mm			
	$f_y$	$f_{sh}$	$f_{sh\ ult}$
Mean	518.1224	530.5282	675.3878
	$\epsilon_y$	$\epsilon_{sh}$	$\epsilon_{sh\ ult}$
Mean	0.002441	0.017151	0.140765
Total of Samples	38		
Diameter of Bars 22 mm			
	$f_y$	$f_{sh}$	$f_{sh\ ult}$
Mean	503.6793	514.2421	665.0723
	$\epsilon_y$	$\epsilon_{sh}$	$\epsilon_{sh\ ult}$
Mean	0.002478	0.016764	0.136128
Total of Samples	40		

Table 3. Section properties of column models

No	A (mm)	B (mm)	C (MPa)	D	E	F	G (mm)	H (MPa)	I (mm)								
1	10	40	20	2	2	D19	100	300	320x320								
2									400x400								
3									500x500								
4									D22	100	300	320x320					
5												400x400					
6												500x500					
7									D19	100	300	400x400	50				
8													100				
9													150				
10													200				
11													D22	100	300	400x400	50
12																	100
13																	150
14																	200
15													D19	100	300	400x400	2
16																	3
17																	4
18																	2
19									D22	100	300	400x400	3				
20													4				
21													D19	100	300	400x400	200
22																	300
23									400								
24									500								
25									D22	100	300	400x400	200				
26													300				
27													400				
28													500				
29									D19	100	300	400x400	D19				
30													D22				

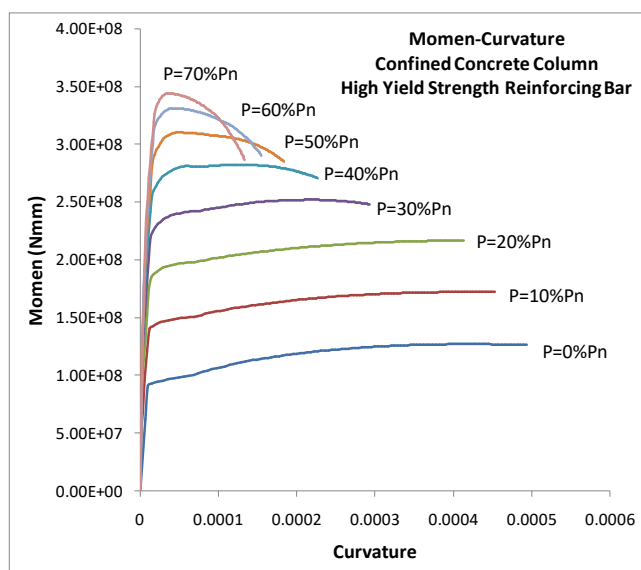


Figure 4. Effect of Axial Load on Moment-Curvature Curve

Table 4 reveal the influence of column area on the column flexural overstrength factor and curvature ductility. Table 5 reveal the influence of transverse reinforcement spacing on the column flexural overstrength factor and curvature ductility. Table 5 reveal the influence of reinforcement ratio on the column flexural overstrength factor and curvature ductility. Table 5 show the effect of transverse bar yield strength on the column flexural overstrength factor and curvature ductility. Table 6 show the effect of concrete compression strength on the column flexural overstrength factor and curvature ductility.

Table 4. Curvature ductility and overstrength factor of column model 1-6

No	width x depth	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean	No	width x depth	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean
1	320x320 mm	0%	27.59774	1.205021	4	320x320 mm	0%	22.61007	1.199156
		10%	20.9468	1.142588			10%	18.38696	1.152273
		20%	16.82654	1.145527			20%	15.05599	1.167069
		30%	11.16828	1.252914			30%	9.83067	1.318216
		40%	8.113128	1.45598			40%	7.939746	1.525626
		50%	7.349483	1.658709			50%	7.250162	1.727583
		60%	6.965344	1.899888			60%	6.848009	1.727583
2	400x400 mm	0%	38.82601	1.239789	5	400x400 mm	0%	27.98066	1.234346
		10%	25.40629	1.141497			10%	23.81144	1.153754
		20%	19.37351	1.131233			20%	16.88271	1.143172
		30%	11.9714	1.159874			30%	10.66488	1.178537
		40%	8.174235	1.3268			40%	7.736116	1.368689
		50%	7.242987	1.516635			50%	7.171104	1.558718
		60%	6.963187	1.743437			60%	6.866014	1.783941
3	500x500 mm	0%	44.57443	1.270774	6	500x500 mm	0%	38.12032	1.267146
		10%	33.17002	1.131175			10%	27.3468	1.146506
		20%	19.08353	1.108993			20%	17.5314	1.118084
		30%	11.57543	1.135569			30%	10.82738	1.136498
		40%	7.910262	1.248385			40%	7.559381	1.270044
		50%	7.12519	1.423261			50%	7.099014	1.448317
		60%	6.943154	1.631732			60%	6.891836	1.659066
70%	6.846582	1.943816	70%	6.78232	1.970891				

Table 5. Curvature ductility and overstrength factor of column model 7 - 28

No	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean	No	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean	No	longitudinal steel Ast (mm <sup>2</sup> )	$\rho$	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean	No	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean	No	P/Pn	$\mu\phi$ mean	$\lambda\phi$ mean
7	0%	68.155	1.349	9	0%	26.012	1.161	15	1134.114948	0.0071	0%	38.82601	1.239789	21	0%	29.25	1.185	23	0%	47.319	1.279
	10%	49.933	1.228		10%	16.198	1.083				10%	19.37351	1.131233		10%	18.492	1.1		10%	31.318	1.172
	20%	40.183	1.217		20%	11.971	1.08				20%	11.9714	1.159874		20%	13.924	1.096		20%	24.415	1.159
	30%	24.907	1.242		30%	7.6062	1.123				30%	8.174235	1.3268		30%	8.4853	1.135		30%	15.256	1.183
	40%	17.561	1.406		40%	6.7602	1.276				40%	7.242987	1.516635		40%	6.4889	1.293		40%	10.361	1.35
	50%	12.701	1.632		50%	6.4356	1.437				50%	6.963187	1.743437		50%	6.1153	1.464		50%	8.0696	1.554
	60%	9.6849	1.926		60%	6.2756	1.614				60%	6.807308	2.086627		60%	5.9204	1.66		60%	7.6903	1.803
8	0%	38.826	1.24	10	0%	19	1.108	16	1701.172422	0.0106	0%	29.64941	1.217909	22	0%	38.826	1.24	24	0%	54.818	1.307
	10%	25.406	1.141		10%	11.196	1.045				10%	23.70118	1.152181		10%	25.406	1.141		10%	37.272	1.195
	20%	19.374	1.131		20%	7.7571	1.051				20%	16.96006	1.138542		20%	19.374	1.131		20%	29.276	1.182
	30%	11.971	1.16		30%	5.2029	1.093				30%	10.48851	1.183428		30%	11.971	1.16		30%	18.487	1.203
	40%	8.1742	1.327		40%	4.7337	1.23				40%	7.798096	1.374991		40%	8.1742	1.327		40%	12.482	1.368
	50%	7.243	1.517		50%	4.5157	1.367				50%	7.221359	1.570685		50%	7.243	1.517		50%	9.0923	1.583
	60%	6.9632	1.743		60%	4.4104	1.464				60%	6.895314	1.801419		60%	6.9632	1.743		60%	7.4018	1.849
11	0%	52.836	1.347	13	0%	18.527	1.151	18	1520.530844	0.0095	0%	6.714223	2.141861	25	0%	21.583	1.177	27	0%	34.294	1.277
	10%	45.725	1.25		10%	14.976	1.088				10%	6.74223	2.141861		10%	17.617	1.108		10%	29.603	1.189
	20%	35.055	1.242		20%	10.592	1.089				20%	27.96766	1.209665		20%	12.49	1.106		20%	21.292	1.173
	30%	22.361	1.271		30%	7.4269	1.142				30%	21.94859	1.161595		30%	7.7086	1.153		30%	13.535	1.203
	40%	15.75	1.453		40%	6.7035	1.317				40%	14.93177	1.138372		40%	6.5412	1.334		40%	9.3148	1.392
	50%	11.624	1.678		50%	6.3509	1.477				50%	9.485997	1.218864		50%	6.1296	1.504		50%	7.9727	1.597
	60%	8.7797	1.974		60%	6.1776	1.642				60%	7.781492	1.416571		60%	5.915	1.694		60%	7.5903	1.846
12	0%	27.981	1.234	14	0%	13.821	1.097	19	2280.796267	0.0143	0%	7.154041	1.613011	26	0%	5.8028	1.939	28	0%	40.69	1.307
	10%	23.811	1.154		10%	10.436	1.046				10%	6.80422	1.842079		10%	27.981	1.234		10%	35.027	1.215
	20%	16.883	1.143		20%	7.0705	1.052				20%	6.611503	2.162602		20%	16.883	1.143		20%	25.699	1.199
	30%	10.665	1.179		30%	5.3328	1.113				30%	26.40738	1.218033		30%	10.665	1.179		30%	16.415	1.226
	40%	7.7361	1.369		40%	4.8132	1.269				40%	21.09245	1.165615		40%	7.7361	1.369		40%	11.417	1.411
	50%	7.1711	1.559		50%	4.5706	1.401				50%	14.41464	1.145421		50%	7.1711	1.559		50%	8.3169	1.627
	60%	6.866	1.784		60%	4.4591	1.47				60%	9.228261	1.230005		60%	6.866	1.784		60%	7.3229	1.894
70%	6.6971	2.123	70%	4.6353	1.616	70%	26.40738	1.218033	70%	6.6971	2.123	70%	7.0475	2.305							

Table 6. Curvature ductility and overstrength factor of column model 29, 30

No	P/Pn	$\mu\phi$ mean	$\lambda_0$ mean
29	0%	31.87612	1.206455
	10%	20.25377	1.086586
	20%	13.79796	1.073527
	30%	8.355373	1.108279
	40%	5.988974	1.218472
	50%	5.712133	1.356078
	60%	5.574538	1.503026
	70%	5.506238	1.69581
No	P/Pn	$\mu\phi$ mean	$\lambda_0$ mean
30	0%	26.89537	1.195343
	10%	19.0761	1.094898
	20%	12.83532	1.083489
	30%	7.670564	1.112439
	40%	6.059716	1.247438
	50%	5.743641	1.386436
	60%	5.585848	1.528571
	70%	5.50762	1.704156

## 6. CONCLUSIONS

The overstrength value decreased at low levels of axial load (P/Pn 0% - 30%) but at higher axial loads (P/Pn > 30%), the ratio of Mmax (experimental flexural strengths of square columns section) to Mi (predictions based on ideal flexural strength) increased as shown by Tables 4, 5, 6, 7, and 8. The Ideal flexural strength is determined by using measured material strengths, an ultimate compression strain of 0.003. The increase in compression zone depth, kd, with axial load, and hence the greater importance of the term Cc (kd -  $\beta.kd/2$ ) to the total flexural strength caused the increased of overstrength factor.

The relationship between axial load and The curvature ductility ( $\mu\phi$ ) is obtained from Tables 4, 5, 6, 7, and 8. It is exhibit that the ductility of the column is significantly reduced by the presence of axial load. The flexural overstrength value for column reinforced steel with yield strength above 500 MPa is 1.04 – 2.30.

The stress-strain curve for high strength reinforcement can be determined by six variable basic parameters ( $f_y, \epsilon_y, E_s, f_{sh}, \epsilon_{sh}, E_{sh}$ ).

There are six key parameters (column area, transverse reinforcement spacing, reinforcement ratio, transverse bars yield strength, concrete compression strength, and axial load) primarily influence the curvature ductility and flexural overstrength factor. The most influencing parameter is found to be the presence of axial load

## REFERENCES

- [1]. ACI, 2011, Building Code Requirements for Structural Concrete and Commentary, ACI 318-11, ACI Committee 318, American Concrete Institute, Farmington Hills, Michigan.
- [2]. Andriono, T., and Park, R., (1986), "Seismic Design Considerations of the Properties of New Zealand Manufactured Steel Reinforcing Bars", Bulletin of the New Zealand National Society for Earthquake Engineering, Vol.19, No. 3, September 1986 [3] Todd, K.D. and Mays, L.W. (2005). *Groundwater Hydrology*, 3rd edition, John Wiley & Sons, Inc., New York.
- [3]. Andriono, T., (1986), Properties of Reinforcing Steel Used In Seismic Design, Report submitted in partial fulfilment of the Requirments for the Degre of Master of Engineering at the University of Canterbury
- [4]. ASTM, 2009a, Standard Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement, ASTM A615-09b, ASTM International, West Conshohocken, Pennsylvania.
- [5]. ASTM, 2009a, Standard Specification for Deformed and Plain Carbon-Steel Bars for Concrete Reinforcement, ASTM A615-09b, ASTM International, West Conshohocken, Pennsylvania.
- [6]. ASTM, 2009b, Standard Specification for Low-Alloy Steel Deformed and Plain Bars for Concrete Reinforcement, ASTM A706-09b, ASTM International, West Conshohocken, Pennsylvania.
- [7]. Lim Wai Tat (1991), Statistical Analysis of Reinforcing Steel Properties, University of Canterbury Christchurch, New Zealand.
- [8]. Kent, D. C., and Park, R., Flexural Members with Confined Concrete, Journal of Structural Division, ASCE, V. 97, No. ST7, July 1971, pp. 1969-1990.

- [9]. Kappos, A. J., and Konstantinidis, D., Statistical Analysis of Confined High-Strength Concrete Columns, *Material and Structures*, V. 32, Dec. 1992, pp. 734-748.
- [10]. Mander, J. B., Priestley, M. J. N., and Park, R. (1984). "Seismic design of bridge piers." Research Rep. 84-02, Dept. of Civ. Engrg., University of Canterbury, Christchurch, New Zealand.
- [11]. Park, R; Paulay, T (1975), *Reinforced Concrete Structures*, John Wiley and Sons, New York, USA
- [12]. Park, R., "Constitutive Relations of Steel : Effect on Strength Consideration in Seismic Design", *Proceedings of Workshop on Earthquake Resistant Reinforced Concrete Building Construction*, Vol.II, University of California, Berkeley, July 1977, pp. 683-695
- [13]. Priestley, M.J.N; Paulay, T (1990), *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley and Sons, , 3rd edition.
- [14]. Standar Nasional Indonesia 03-1726-2002, *Tata Cara Perencanaan KetahananGempa untuk Bangunan Gedung*, Badan Standardisasi Nasional , Jakarta.
- [15]. Standar Nasional Indonesia.SNI-03-2847-2002 - *Tata Cara Perhitungan Struktur Beton Untuk Bangunan Gedung*, Standar Nasional Indonesia.
- [16]. Susanti Eka (2012), "Kemampuan Daktilitas Baja Tulangan Dengan Mutu Diatas 400 MPa Untuk Disain Struktur Tahan Gempa", *Seminar Nasional Pascasarjana XII - ITS*, Surabaya.